Ratios of Polygonal Numbers as Continued Fractions

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Abstract: Representations of rational numbers as continued fraction always exist. In number theory study of polygonal numbers has various approaches. Here in this paper we identify the patterns of continued fractions of ratios of polygonal numbers of consecutive order.

Keywords: Continued fractions, Simple continued fraction, Euclidean algorithm, Square numbers, Hexagonal numbers, Centered Hexagonal numbers, Star numbers.

Notations:
1. \(\langle a_0, a_1, a_2, a_3, \ldots, a_n \rangle\) : Continued fraction expansion.
2. \(\left\lfloor \frac{n}{2} \right\rfloor\) : Integer part of the rational number \(n/2\).
3. \(T_n = \frac{n(n+1)}{2}\) : \(n^{a}\) Triangular number
4. \(S_n = n^2\) : \(n^{a}\) Square number
5. \(H_n = 3n(n-1) + 1\) : \(n^{a}\) Centered Hexagonal number
6. \(S^*_n = 6n(n-1) + 1\) : \(n^{a}\) Star number

1. Introduction

The Indian mathematician Aryabhata used a continued fraction to solve a linear indeterminate equation. For more than a thousand years, any work that used continued fractions was restricted to specific examples. Throughout Greek and Arab mathematical writing, we can find examples and traces of continued fractions. Euler showed that every rational can be expressed as a terminating simple continued fraction. He also provided an expression for \(e\) in continued fraction form. He used this expression to show that \(e\) and \(e^2\) are irrational [3].

Polygonal numbers have graphical representation. Golden ratio which is an irrational also has a graphical representation. This idea moti vated me to create a set of rational numbers using polygonal numbers and represent them in terms of continued
fractions. First we give different representations of a rational number as a continued fraction [2, 3, 4, 5].

An expression of the form
\[
\frac{p}{q} = a_0 + \frac{b_0}{a_1 + \frac{b_1}{a_2 + \frac{b_2}{a_3 + \cdots}}}
\]

Where \(a_i, b_i\) are real or complex numbers is called a continued fraction.

An expression of the form
\[
\frac{p}{q} = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}
\]

Where \(b_i = 1 \forall i\), and \(a_i, a_i, a_i, \cdots\) are each positive integers also represents a simple continued fraction.

The continued fraction is commonly expressed as
\[
\left\langle a_0, a_1, a_2, a_3, \cdots \right\rangle
\]

The elements \(a_0, a_1, a_2, a_3, \cdots\) are called the partial quotients. If there are finite number of partial quotients, we call it finite simple continued fraction, otherwise it is infinite. We have to use either Euclidean algorithm[1] or continued fraction algorithm to find such partial quotients. The finite simple continued fraction is denoted by \(\left\langle a_0, a_1, a_2, a_3, \cdots a_n \right\rangle\) and it has an alternate form \(\left\langle a_0, a_1, a_2, a_3, \cdots a_n, 1, 1 \right\rangle\).

1.1 The Continued Fraction Algorithm

Suppose we wish to find continued fraction expansion of \(x \in \mathbb{R}\).

Let \(x_o = x\) and set \(a_o = \lfloor x_o \rfloor\), Define \(x_o = x_o - \lfloor x_o \rfloor \) and set \(a_o = \lfloor x_o \rfloor\)

and \(x_{i-1} = x_{i-1} - \lfloor x_{i-1} \rfloor\) \(\Rightarrow a_{i-1} = \lfloor x_{i-1} \rfloor\). \(\Rightarrow x_i = \frac{1}{x_{i-1} - \lfloor x_{i-1} \rfloor}\) \(\Rightarrow a_i = \lfloor x_i \rfloor\). This process is continued infinitely or to some finite stage till an \(x_i \in \mathbb{N}\) exists such that \(a_i = \lfloor x_i \rfloor\).
2. Triangular Numbers

Definition 2.1[5, 6]: Triangular Numbers

The numbers 1, 3, 6, 10, 15, 21, 28, 36, 45, …, \( T_n = \frac{n(n+1)}{2} \), … are called triangular numbers, since the \( n \)th number counts the number of dots in an equilateral triangular array with \( n \) dots to the side.

\[
T_1 = 1 \quad T_2 = 3 \quad T_3 = 6 \quad T_4 = 10 \quad T_5 = 15 \quad T_6 = 21
\]

Theorem 2.1:

For \( n \geq 3 \), \( \frac{T_n}{T_{n+1}} = \begin{cases} 
\langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 2 \rangle & \text{when } n \text{ is odd.} \\
\langle 0; 1, \frac{n}{2} \rangle & \text{when } n \text{ is even.}
\end{cases} 
\)

Proof:

Using algorithm 1.1 the proof follows

Since \( T_n = \frac{n(n+1)}{2} \) and \( T_{n+1} = \frac{(n+1)(n+2)}{2} \),

\[
\frac{T_n}{T_{n+1}} = \frac{n}{n+2}
\]

When \( n \) is even,

\[
\frac{T_n}{T_{n+1}} = \frac{n}{n+2} = 0 + \frac{1}{\frac{n+2}{n}} = 0 + \frac{1}{1 + \frac{n}{2}} = 0 + \frac{1}{1 + \frac{n}{2}}.
\]

When \( n \) is odd,

\[
\frac{T_n}{T_{n+1}} = \frac{n}{n+2} = 0 + \frac{1}{\frac{n+2}{n}} = 0 + \frac{1}{1 + \frac{2}{n}} = 0 + \frac{1}{1 + \frac{2}{n}}.
\]

Hence for each \( n \geq 3 \), the continued fraction expansion of \( \frac{T_n}{T_{n+1}} \) is \( \langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 2 \rangle \) when \( n \) is even and is \( \langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 2 \rangle \) when \( n \) is odd.
3. Square Numbers

Definition 3.1[5, 6]: Square Numbers

A square number or perfect square is an integer that is the square of an integer or it is the product of some integer with itself.

In other words, the numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, … are called square numbers, since the nth number counts the number of dots in a square array with n dots to each side.

\[ S_1 = 1 \quad S_2 = 4 \quad S_3 = 9 \quad S_4 = 16 \quad S_5 = 25 \]

---

Theorem: 3.1:

For \( n \geq 3 \),

\[
\frac{S_n}{S_{n+1}} = \begin{cases} 
\langle 0;1, \left\lfloor \frac{n}{2} \right\rfloor, 3, 1, \left\lfloor \frac{n}{2} \right\rfloor \rangle & \text{when } n \text{ is odd.} \\
\langle 0;1, \frac{n-2}{2}, 1, 1, \frac{n}{2} \rangle & \text{when } n \text{ is even.}
\end{cases}
\]

Proof:

Using algorithm 1.1 the proof follows

Case (i): \( n \) is odd

Taking \( n = 3 \), \( \frac{S_3}{S_4} = \frac{9}{16} \)

Let \( x_o = \frac{9}{16} \), so \( a_0 = 0 \).

Then

\[
x_i = \frac{1}{x_{i-1} - \left\lfloor x_{i-1} \right\rfloor} = \frac{1}{9} = 1 + \frac{7}{9} \Rightarrow a_i = 1.
\]

\[
x_3 = \frac{1}{x_2 - \left\lfloor x_2 \right\rfloor} = \frac{7}{2} = 3 + \frac{1}{2} \Rightarrow a_3 = 3.
\]

\[
x_4 = \frac{1}{x_3 - \left\lfloor x_3 \right\rfloor} = \frac{2}{1} = 2 \Rightarrow a_4 = 2.
\]

Therefore \( \frac{9}{16} = \langle 0;1,1,3,2 \rangle \) or \( \frac{9}{16} = \langle 0;1,1,1,1,1 \rangle \).
Hence $\frac{S_i}{S_i} = \langle 0; 1, 1, 3, 1, 1 \rangle$. The result is true when $n = 3$. The result is true for $n = 2k - 1$, where $3 \leq k \leq n$.

Then, $\frac{S_{2k-1}}{S_{2k}} = \langle 0; 1, k - 1, 3, 1, k - 1 \rangle$

We prove the result for $n = 2k + 1$.

Here we have to find the continued fraction of $\frac{S_{2k+1}}{S_{2k+2}} = \frac{(2k + 1)^2}{(2k + 2)^2}$.

Let $x_o = \frac{(2k + 1)^2}{(2k + 2)^2} = \frac{4k^2 + 4k + 1}{4k^2 + 8k + 4}$, so $a_o = 0$.

Then $x_i = \frac{1}{x_i - \lfloor x_i \rfloor} = \frac{4k^2 + 8k + 4}{4k^2 + 4k + 1} = 1 + \frac{4k + 3}{4k + 4} \Rightarrow a_i = 1$.

Since $x_i = k$, $\Rightarrow S_{2k+1} = \langle 0; 1, k, 3, 1, k \rangle$.

Hence by induction the result is true for all values of $n$ where $n$ is odd.

Case(ii): $n$ is even

Taking $n = 4$, $\frac{S_i}{S_i} = \frac{16}{25}$

Let $x_o = \frac{16}{25}$, so $a_o = 0$.

Then $x_i = \frac{1}{x_i - \lfloor x_i \rfloor} = \frac{25}{16} = 1 + \frac{9}{16} \Rightarrow a_i = 1$. 

$\Rightarrow S_{2k+1} = \langle 0; 1, k, 3, 1, k \rangle$. 

Hence by induction the result is true for all values of $n$ where $n$ is odd.
\[
x_1 = \frac{1}{x_0 - [x_0]} = \frac{9}{7} = 1 + \frac{2}{7} \Rightarrow a_1 = 1.
\]
\[
x_2 = \frac{1}{x_1 - [x_1]} = \frac{7}{2} = 3 + \frac{1}{2} \Rightarrow a_2 = 3.
\]
\[
x_3 = \frac{1}{x_2 - [x_2]} = 2 \Rightarrow a_3 = 2.
\]
Since \( x_3 = 2 \), \( \Rightarrow S_x = \langle 0; 1, 1, 3, 2 \rangle \).

The result is true when \( n = 4 \). The result is true for \( n = 2k - 2 \), where \( 3 \leq k \leq n \).

Then, \( \frac{S_{2k-2}}{S_{2k-1}} = \langle 0; 1, k - 2, 1, 3, k - 1 \rangle \).

We prove the result for \( n = 2k \).

Here we have to find the continued fraction of \( \frac{S_{2k}}{S_{2k+1}} = \frac{(2k)^2}{(2k+1)^2} \).

Let \( x_0 = \frac{(2k)^2}{(2k+1)^2} \), so \( a_0 = 0 \).

Then \( x_1 = \frac{1}{x_0 - [x_0]} = \frac{4k^2 + 4k + 1}{4k^2} = 1 + \frac{4k + 1}{4k^2} \Rightarrow a_1 = 1. \)
\( x_2 = \frac{1}{x_1 - [x_1]} = \frac{4k^2 + 4k + 1}{4k + 1} = k + \frac{3k + 1}{4k + 1} \Rightarrow a_2 = k - 1. \)
\( x_3 = \frac{1}{x_2 - [x_2]} = \frac{4k + 1}{3k + 1} = 1 + \frac{k}{3k + 1} \Rightarrow a_3 = 1. \)
\( x_4 = \frac{1}{x_3 - [x_3]} = \frac{3k + 1}{k} = 3 + \frac{1}{k} \Rightarrow a_4 = 3. \)
\( x_5 = \frac{1}{x_4 - [x_4]} = k \Rightarrow a_5 = k. \)

Since \( x_5 = k \), \( \Rightarrow S_{2k} = \langle 0; 1, k - 1, 1, 3, k \rangle \).

Hence by induction the result is true for all values of \( n \) where \( n \) is even. Hence from case(i) and (ii) we have. For each \( n \geq 3 \), the continued fraction expansion of
\[ \frac{S_n}{S_{n+1}} \text{ is } \langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 3, 1, \left\lfloor \frac{n}{2} \right\rfloor \rangle \text{ when } n \text{ is odd and is } \langle 0; 1, \frac{n}{2} - 1, 1, 3, \frac{n}{2} \rangle \text{ when } n \text{ is even.} \]

4. Centered Hexagonal Number

Definition 4.1[7]: Centered Hexagonal Number

A centered hexagonal number, or hex number, is a centered figurate number that represents a hexagon with a dot in the center and all other dots surrounding the center dot in a hexagonal lattice.

The \( n \)th centered hexagonal number is given by the formula

\[ H_n = 3n(n - 1) + 1 \]

Another way of expressing the hex number as

\[ H_n = 6(n - 1) + 1 \]

Shows that the centered hexagonal number for \( n \) is 1 more than 6 times the \((n - 1)\)th triangular number. The first few centered hexagonal numbers are

1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, 469, 547, 631, 721, 817, 919

\[ H_1 = 1 \quad H_2 = 7 \quad H_3 = 19 \quad H_4 = 37 \]

Theorem: 4.1:

For \( n \geq 3 \),

\[ \frac{H_i}{H_{i+1}} = \begin{cases} \langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 6n \rangle & \text{ when } n \text{ is odd.} \\ \langle 0; 1, \frac{n}{2} - 1, 1, 1, \frac{3n}{2} - 1, 2 \rangle & \text{ when } n \text{ is even.} \end{cases} \]

Proof:

Using algorithm 1.1 the proof follows

Case (i): \( n \) is odd

Taking \( n = 3 \),

\[ \frac{H_j}{H_i} = \frac{19}{37} \]

Let \( x_0 = \frac{19}{37} \), so \( a_0 = 0 \).
Then \[ x_1 = \frac{1}{x_0 - \lfloor x_0 \rfloor} = \frac{37}{19} = 1 + \frac{18}{19} \Rightarrow a_1 = 1. \]
\[ x_2 = \frac{1}{x_1 - \lfloor x_1 \rfloor} = \frac{19}{18} = 1 + \frac{1}{18} \Rightarrow a_2 = 1. \]
\[ x_3 = \frac{1}{x_2 - \lfloor x_2 \rfloor} = 18 \Rightarrow a_3 = 18. \]
Therefore \[ \frac{19}{37} = \langle 0; 1, 1 \rangle \]

Hence \[ \frac{H_1}{H_2} = \langle 0; 1, 1, 18 \rangle. \]

The result is true when \( n = 3 \). The result is true for \( n = 2k - 1 \), where \( 3 \leq k \leq n \).

Then, \[ H_{2k-1} = \langle 0; 1, k - 1, 6(2k - 1) \rangle. \]

We prove the result for \( n = 2k + 1 \).

Here we have to find the continued fraction of \[ \frac{H_{2k+1}}{H_{2k+2}} = \frac{12k^2 + 6k + 1}{12k^2 + 18k + 7}. \]

Let \[ x_0 = \frac{(2k + 1)^2}{(2k + 2)^2} = \frac{12k^2 + 6k + 1}{12k^2 + 18k + 7}, \text{ so } a_0 = 0. \]

Then \[ x_1 = \frac{1}{x_0 - \lfloor x_0 \rfloor} = \frac{12k^2 + 18k + 7}{12k^2 + 6k + 1} = 1 + \frac{12k + 6}{12k^2 + 6k + 1} \Rightarrow a_1 = 1. \]
\[ x_2 = \frac{1}{x_1 - \lfloor x_1 \rfloor} = \frac{12k^2 + 6k + 1}{12k + 6} = k + \frac{1}{12k + 6} \Rightarrow a_2 = k. \]
\[ x_3 = \frac{1}{x_2 - \lfloor x_2 \rfloor} = 12k + 6 \Rightarrow a_3 = 12k + 6. \]

Since \( x_3 = 12k + 6 \), \[ \frac{H_{2k+1}}{H_{2k+2}} = \langle 0; 1, k, 6(2k + 1) \rangle. \]

Hence by induction the result is true for all values of \( n \) where \( n \) is odd.

Case(ii): \( n \) is even. Taking \( n = 4 \) then \[ \frac{H_4}{H_5} = \frac{37}{61} \]

Let \[ x_0 = \frac{37}{61}, \text{ so } a_0 = 0. \]
Then
\[
x_j = \frac{1}{x_o - \left[ x_o \right]} = \frac{61}{37} = 1 + \frac{24}{37} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_i - \left[ x_i \right]} = \frac{37}{13} = 1 + \frac{11}{24} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_j - \left[ x_j \right]} = \frac{24}{11} = 1 + \frac{11}{13} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_i - \left[ x_i \right]} = \frac{11}{2} = 5 + \frac{1}{11} \implies a_j = 5.
\]
\[
x_j = \frac{1}{x_s - \left[ x_s \right]} = 2 \implies a_j = 2.
\]

Since \[x_j = 2 \implies \frac{H_j}{H_4} = \langle 0; 1, 1, 1, 1, 5, 2 \rangle.\]

The result is true when \[n = 4.\] The result is true for \[n = 2k - 2,\] where \[3 \leq k \leq n.\]

Then, \[\frac{H_{2n+1}}{H_{2n-1}} = \langle 0; 1, k - 2, 1, 1, 3k - 4, 2 \rangle.\]

We prove the result for \[n = 2k.\]

Here we have to find the continued fraction of \[\frac{H_{2n}}{H_{2n+1}} = \frac{12k^2 - 6k + 1}{12k^2 + 6k + 1}.\]

Let \[x_o = \frac{(2k)^2}{(2k + 1)^2} = \frac{12k^2 - 6k + 1}{12k^2 + 6k + 1},\] so \[a_o = 0.\]

Then
\[
x_j = \frac{1}{x_o - \left[ x_o \right]} = \frac{12k^2 + 6k + 1}{12k^2 - 6k + 1} = 1 + \frac{12k}{12k^2 - 6k + 1} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_i - \left[ x_i \right]} = \frac{12k^2 - 6k + 1}{12k} = k - 1 + \frac{6k + 1}{12k} \implies a_j = k - 1.
\]
\[
x_j = \frac{1}{x_j - \left[ x_j \right]} = \frac{12k}{6k + 1} = 1 + \frac{6k - 1}{6k + 1} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_i - \left[ x_i \right]} = \frac{6k + 1}{6k - 1} = 1 + \frac{2}{6k - 1} \implies a_j = 1.
\]
\[
x_j = \frac{1}{x_s - \left[ x_s \right]} = \frac{6k - 1}{2} = 3k - 1 + \frac{1}{2} \implies a_j = 3k - 1.
\[ x_e = \frac{1}{x_s} - \left\lfloor \frac{x_s}{2} \right\rfloor = 2 \Rightarrow a_e = 2. \]

Since \( x_e = 2 \),
\[ \Rightarrow \frac{H_{2e}}{H_{2e+3}} = \left\langle 0; 1, k+1, 1, 3k+1, 2 \right\rangle. \]

Hence by induction the result is true for all values of \( n \) where \( n \) is even.
Hence from case(i) and (ii) we have For each \( n \geq 3 \), the continued fraction expansion of
\[ \frac{H_n}{H_{n+1}} \] is \( \left\langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor \right. \) when \( n \) is odd and is \( \left\langle 0; 1, \frac{n}{2}-1, 1, 1, 3n-1, 2 \right\rangle \) when \( n \) is even.

5. Star Numbers

Definition 5.1[7]: Star Numbers
A star number is a centered figurate number that represents a centered hexagram (six-pointed star), such as the one that Chinese checkers is played on.

The \( n \)th star number is given by the formula
\[ S_n^* = 6n(n-1) + 1 \]

The first few star numbers are 1, 13, 37, 73, 121, 181, 253, 337, 433, 541
The digital root of a star number is always 1 or 4, and progresses in the sequence 1, 4, 1.

\[ S_1^* = 1 \quad S_2^* = 13 \quad S_3^* = 37 \]

Theorem: 5.1:

For \( n \geq 3 \),
\[ \frac{S_n^*}{S_{n+1}^*} = \begin{cases} \left\langle 0; 1, \left\lfloor \frac{n}{2} \right\rfloor, 12n \right\rangle & \text{when } n \text{ is odd.} \\ \left\langle 0; 1, \frac{n}{2}-1, 1, 1, 3n-1, 2 \right\rangle & \text{when } n \text{ is even.} \end{cases} \]

Proof: Similar to the proof of theorem 2
6. The following table summarizes the continued fraction expansion of consecutive fractions of some polygonal numbers.

<table>
<thead>
<tr>
<th>Types of Numbers</th>
<th>Consecutive Fraction of Numbers</th>
<th>Continued Fractions</th>
<th>Nature of ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular number</td>
<td>( \frac{T_n}{T_{n+1}} )</td>
<td>( \langle 0;1,\left[ \frac{n}{2} \right],2 \rangle )</td>
<td>Odd</td>
</tr>
<tr>
<td>Triangular number</td>
<td>( \frac{T_n}{T_{n+1}} )</td>
<td>( \langle 0;1,\frac{n}{2} \rangle )</td>
<td>Even</td>
</tr>
<tr>
<td>Square number</td>
<td>( \frac{S_n}{S_{n+1}} )</td>
<td>( \langle 0;1,\left[ \frac{n}{2} \right],1,\left[ \frac{n}{2} \right] \rangle )</td>
<td>Odd</td>
</tr>
<tr>
<td>Square number</td>
<td>( \frac{S_n}{S_{n+1}} )</td>
<td>( \langle 0;1,\frac{n}{2} \rangle )</td>
<td>Even</td>
</tr>
<tr>
<td>Centered Hexagonal Number</td>
<td>( \frac{H_n}{H_{n+1}} )</td>
<td>( \langle 0;1,\left[ \frac{n}{6} \right],6n \rangle )</td>
<td>Odd</td>
</tr>
<tr>
<td>Centered Hexagonal Number</td>
<td>( \frac{H_n}{H_{n+1}} )</td>
<td>( \langle 0;1,\frac{n}{2} \rangle )</td>
<td>Even</td>
</tr>
<tr>
<td>Star Number</td>
<td>( \frac{S^<em>}{S^</em>_{n+1}} )</td>
<td>( \langle 0;1,\left[ \frac{n}{12} \right],12n \rangle )</td>
<td>Odd</td>
</tr>
<tr>
<td>Star Number</td>
<td>( \frac{S^<em>}{S^</em>_{n+1}} )</td>
<td>( \langle 0;1,\frac{n}{2} \rangle )</td>
<td>Even</td>
</tr>
</tbody>
</table>

7. Illustration
The following table gives the patterns of continued fraction of consecutive fractions of some polygonal numbers.

<table>
<thead>
<tr>
<th>Consecutive fraction of numbers</th>
<th>Continued fraction expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{T_n}{T_{n+1}} )</td>
<td>( \langle 0;1, 3, 2 \rangle )</td>
</tr>
<tr>
<td>( \frac{T_m}{T_{m+1}} )</td>
<td>( \langle 0;1, 47 \rangle )</td>
</tr>
<tr>
<td>( \frac{S_{s+1}}{S_{s+1}} )</td>
<td>( \langle 0;1, 4, 1, 3, 5 \rangle )</td>
</tr>
</tbody>
</table>
8. Conclusion

In this paper we have identified various patterns of continued fractions of ratios of polygonal numbers of consecutive order and rank. This work may be extended to higher order figurate numbers like pyramidal numbers.

References:


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