Further Results on Chromatic Number with Complementary Connected Perfect Domination Number of a Graph

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Abstract: The concept of Complementary connected perfect domination number was introduced by G. Mahadevan et.al., in [5]. A subset S of V of a non trivial graph G is said to be complementary connected perfect dominating set if S is a dominating set and <V−S> is connected and has a perfect matching. The minimum cardinality taken over all complementary connected perfect dominating sets in G (CCPD-set) is called the complementary connected perfect domination number of G and is denoted by \( \gamma_{ccp} \). In [6, the authors already characterized the extremal graphs whose sum of complementary connected domination number and chromatic number upto 2n-5. Since the characterization of extremal graphs whose sum of complementary connected domination number and chromatic number equals to 2n-6 for any n > 3

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1. Introduction

By a graph G = (V, E) simple undirected connected graph. The concept of Complementary connected perfect domination number was introduced by G. Mahadevan et.al., in [5]. The A subset S of V of a non trivial graph G is said to be complementary connected perfect dominating set if S is a dominating set and <V−S> has a perfect matching and connected. The minimum cardinality taken over all Complementary connected perfect dominating sets in G (CCPD-set) is called the complementary connected perfect domination number of G and is denoted by \( \gamma_{ccp} \). The minimum number of colours required to colour all the vertices in such a way that the adjacent vertices do not receive the same colour is called the chromatic number and is denoted by \( \chi \).
We use the following notations in our further discussions.

**Notation 1.1:** Let $H$ be a regular graph.

a) $H^k$ is a graph obtained from $H$ by attaching $m$ times an end vertex of $P_k$ to a vertex of $H$.

b) $H^i (m_1, m_2, ..., m_n)$ is a graph obtained from $H$ by attaching $m_i$ pendant edges to the vertex $v_i$, $1 \leq i \leq n$.

c) $H^i (m_1 P_{k_1}, m_2 P_{k_2}, ..., m_n P_{k_n})$ is a graph obtained from $H$ by attaching $m_i$ times an end vertex of a path $P_{k_i}$ on $k_i$ vertices to the vertex $v_i$, $1 \leq i \leq n$.

d) $H^i (u(P_{n}, P_{m}), m_1 P_{k_1}, m_2 P_{k_2}, ..., m_n P_{k_n})$ is the graph obtained from $H$ by attaching an end vertex of $P_n$ and an end vertex of $P_m$ to a vertex $u=v_i$ of $H$ and attaching the $m_i$ times an end vertex of $P_{k_i}$ to the vertex $v_i$, $2 \leq i \leq n$.

**Notation 1.2:** $P_n^i (u(P_{n}, P_{m}))$ is the graph obtained from $P_n$ by attaching an end vertex of $P_n$ and an end vertex of $P_m$ to an end vertex $u$ of $P_k$.

**Notation 1.3:** $P_n^i (m_1 P_{k_1}, n_1 P_{s_1})$ is the graph obtained from $P_n$ by attaching $m_1$ times an end vertex of $P_{k_1}$ to an end vertex of $P_n$ and by attaching $n_1$ times an end vertex of $P_{s_1}$ to the other end vertex of $P_n$.

**Notation 1.4:** $P_n^i (U_n, C_s)$ is the graph by attaching one vertex of $C_s$ and one vertex of $C_s$ to the end vertices of $P_n$.

**Notation 1.5:** $P_n^i (m_1 P_{k_1}, C_s)$ is the graph by attaching $m_1$ times of $P_{k_1}$ to an end vertex of $P_n$ and attaching a vertex of $C_s$ to other end vertex of $P_n$.

**Theorem 1.6:** For any graph $G$, $\gamma_{ccp}(G) = n$ if and only if $G$ is a star.

**Theorem 1.7:** Let $G$ be a connected graph with $\gamma_{ccp} = n - 2$ and $\chi = n - 4$. Then $\gamma_{ccp} + \chi = 2n - 6$, for any $n > 3$. If and only if $G$ is isomorphic to $G_1 = \{C_4(P_3), C_6(P_2)\}$ $P_4, C_6, P_4(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0)$, $P_4(2P_2), P_4(2P_2), C_6(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0), C_6(u(P_3, P_3), 0)$. $C_6(u(P_3, P_3), 0), P_4(2P_2), P_4(2P_2), C_6(3P_2, C_3), C_6(3P_2, C_3), K_4(4, 0, 0, 0), K_4(3, 1, 0, 0), K_4(2, 2, 0, 0), K_4(1, 1, 1, 1), K_4(4, 0, 0), K_4(2, 2, 0), K_4(3, 1, 0)$ and any one of the following figure 1.1.
2. Main Result

In [6], it has been already characterized the extremal graphs whose sum of complementary connected domination number and chromatic number up to $2n-5$. Since the characterization of graphs whose sum of complementary connected perfect domination number and chromatic number is equal to $2n-6$.

**Theorem 2.1** Let $G$ be a connected graph with $\gamma_{ccp} = n - 4$ and $\chi = n - 2$. Then $\gamma_{ccp} + \chi = 2n - 6$, for any $n > 3$ if and only if $G$ is isomorphic to $K_6(2P_2)$, $K_6(1,1,0,0,0,0)$, $K_5(2P_2)$, $K_5(1,1,0,0,0)$, $K_4(P_3)$, $G_i$ and any of the following graphs in Figure 1.2.
Proof: Let $\gamma_{cep}(G) + \chi(G) = 2n - 6$, then $\gamma_{cep} + \chi = 2n - 6$ for any $n > 3$. Then all the possible cases are (i) $\gamma_{cep} = n$ and $\chi = n - 6$, (ii) $\gamma_{cep} = n - 1$ and $\chi = n - 5$, (iii) $\gamma_{cep} = n - 2$ and $\chi = n - 4$, (iv) $\gamma_{cep} = n - 3$ and $\chi = n - 3$, (v) $\gamma_{cep} = n - 4$ and $\chi = n - 2$, (vi) $\gamma_{cep} = n - 5$ and $\chi = n - 1$, (vii) $\gamma_{cep} = n - 6$ and $\chi = n$. 

Figure 1.2
The cases, (ii), (iv), (vi) \( < V \sim S > \) has odd number of vertices. Hence, it not possible to form a perfect matching. Hence in the all these cases, no graph exists. For the remaining cases, the graph exists. The case, (iii) already proved in [6].

Case (i) : \( \gamma_{ccp} = n \) and \( \chi = n - 6 \)

Let \( \gamma_{ccp} = n \), by theorem 1.3, G is a star. But for star \( \chi = 2 \) so that \( n = 8 \). Hence \( G = K_{1,7} \).

Case (ii): \( \gamma_{ccp} = n - 4 \) and \( \chi = n - 2 \)

Since \( \chi = n - 2 \), G contains a clique K on \( n = 2 \) or does not contain a clique K on \( n = 2 \) vertices.. Let G contains a clique on K= \( K_{n-2} \) vertices and let \( S= \{v_1, v_2\} \in V(G) - V(K) \). Then the induced sub graph \(<S> = \overline{K_2}, K_1\)

Sub case 1: \(<S> = \overline{K_2}\)

Let \( v_1 \) and \( v_2 \) be the vertices of \( \overline{K_2} \) (i) If \( v_1 \) or \( v_2 \) is mapped to a single vertex say \( u_1 \) of \( K_{n-2} \). (ii) If \( v_1 \) and \( v_2 \) is mapped to different vertices of \( K_{n-2} \).

a) Suppose \( K = K_{n-2} \) has even number of vertices, then \( \{v_1, v_2, u_1, u_2\} \) forms a \( \gamma_{ccp} \) set of G. Since \( \gamma_{ccp} = n - 2 \) so that \( n = 8 \) and hence \( K = K_6 \). In this case the possible graphs are \( K_6(2P_3), K_6(P_3, P_0, 0, 0), K_6(1, 1, 0, 0, 0, 0) \). If \( d(v_1) > 1 \), then we get a contradiction to the hypothesis.

b) Suppose \( K = K_{n-2} \) has odd number of vertices \( \{v_1, v_2, u_1\} \) forms a \( \gamma_{ccp} \) set of G. Since \( \gamma_{ccp} = n - 2 \) so that \( n = 7 \) and hence \( K = K_5 \). Let \( u_1 \) adjacent to \( v_1 \) of \( K_5 \) then the possible graphs are \( K_5(2P_2), K_5(P_2, P_0, 0, 0) \). If \( v_1 \) and \( v_2 \) is adjacent to \( u_1 \) \( d(v_1) = 2 \) and \( d(v_2) = 1 \), then \( G \cong G_i \). If \( d(v_1) = 3 \) and \( d(v_2) = 1 \), then \( G \cong G_2 \). If \( d(v_1) = 4 \) and \( d(v_2) = 1 \), then \( G \cong G_3 \). If \( v_1 \) and \( v_2 \) is adjacent to \( u_1 \) \( d(v_1) = 1 \), then \( G \cong G_4 \). If \( d(v_1) = d(v_2) = 2 \), then \( G \cong G_5 \).

Sub case 2: \(<S> = K_2\)

Let \( v_1 \) and \( v_2 \) be the vertices of \( K_2 \) (i) If \( v_1 \) or \( v_2 \) is mapped to a single vertex say \( u_i \) of \( K_{n-2} \). (ii) If \( v_1 \) and \( v_2 \) is mapped to different vertices of \( K_{n-2} \).

a) Suppose \( K = K_{n-2} \) has even number of vertices, then \( \{v_1, u_1\} \) forms a \( \gamma_{ccp} \) set of G. Since \( \gamma_{ccp} = n - 2 \), we have \( n = 6 \) and hence \( K = K_4 \). In case (i) the possible graphs are \( K_4(P_2), \) and if \( v_1 \) is adjacent to \( u_1 \) \( d(v_1) = 3 \), \( d(v_2) = 1 \), then \( G \cong G_i \). If \( d(v_1) = 4 \), \( d(v_2) = 1 \), then \( G \cong G_4 \). If \( v_1 \) and \( v_2 \) is adjacent to \( u_1 \) and if \( d(v_1) = d(v_2) = 2 \), then
G \cong G_7. If d(v_1) = 3, d(v_2) = 2, then G \cong G_10. If d(v_1) = 4, d(v_2) = 2, then G \cong G_9.

If v_1 is adjacent to u_1 and v_2 is adjacent to u_2, d(v_1) = d(v_2) = 2, then G \cong G_{14}. If d(v_1) = 3, d(v_2) = 2, then G \cong G_{11}. If d(v_1) = 4, d(v_2) = 2, then G \cong G_{12}. If d(v_1) = d(v_2) = 3, then G \cong G_{13}.

b) Suppose K = K_{n-2} has an odd number of vertices, then \{v_1,u_j,u_k\} forms a \gamma_{ccp}-set of G. Since \gamma_{ccp} = n - 2, we have n = 7 and hence K = K_5. Let u_j be adjacent to v_1 of K_5 then the possible graph is K_5(P_3). If v_1 is adjacent to u_j and d(v_1) = 3, then G \cong G_{15}. If d(v_1) = 4, then G \cong G_{16}. If d(v_1) = 5, then G \cong G_{17}. If v_1 is adjacent to u_1 and u_2 and d(v_1) = d(v_2) = 2, then G \cong G_{18}. If v_1 is adjacent to u_1 and v_2 is adjacent to u_2 and d(v_1) = d(v_2) = 2, then G \cong G_{19}. If d(v_1) = 3, d(v_2) = 2, then G \cong G_{20}. If d(v_1) = 4, d(v_2) = 2, then G \cong G_{21}. If d(v_1) = d(v_2) = 3, then G \cong G_{22}.

Case (iii): \gamma_{ccp} = n-6 and \chi = n.

Since \gamma = n, G is a complete graph. If n is even then \gamma_{ccp} = 2 which gives n = 8 and hence G = K_8. If n is odd then \gamma_{ccp} = 1 which gives n = 7 and hence G = K_7. The converse is obvious.

3. Conclusion

In this paper, we characterized the concept of complementary connected perfect domination number and chromatic number equals to 2n-6 for any n > 3 of a graph. The authors are also characterized the sum of complementary connected perfect domination number and chromatic number equals to 2p-7, 2p-8 which will be report in the subsequent papers.

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